Matrix Algebra Tutor - Worksheet 2

Adding Matrices, Subtracting Matrices and Multiplying Matrices by a Scalar

Matrix Algebra Tutor - Worksheet 2 – Adding Matrices, Subtracting Matrices and Multiplying Matrices by a Scalar

1. Suppose you are given two matrices:

$$A = \begin{bmatrix} 3 & 5 & 2 \\ 2 & 8 & 9 \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 & 6 & 2 \\ 6 & 1 & 8 \end{bmatrix}$$

What is $A + B$?

2. Suppose you are given two matrices:

$$A = \begin{bmatrix} 13 & 25 & 12 \\ 22 & 38 & 19 \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 & 16 & 8 \\ 16 & 21 & 7 \end{bmatrix}$$

What is $A - B$?

3. Suppose you are given two matrices:

$$C = \begin{bmatrix} -34 & 25 & -42 \\ 62 & -18 & 29 \end{bmatrix} \text{ and } D = \begin{bmatrix} 17 & 26 & 42 \\ 36 & 31 & 58 \end{bmatrix}$$

What is $C + D$?

4. Suppose you are given two matrices:

$$C = \begin{bmatrix} -32 & -51 & 22 \\ 19 & -81 & -39 \end{bmatrix} \text{ and } D = \begin{bmatrix} -7 & -6 & -8 \\ 26 & 31 & 47 \end{bmatrix}$$

What is $C - D$?

5. Suppose you are given the matrix:

$$E = \begin{bmatrix} 7 & 3 & 2 \\ 8 & 5 & 9 \end{bmatrix}$$

What is $6 \cdot E$?

6. Suppose you are given the matrix:

$$F = \begin{bmatrix} 17 & 26 & -18 \\ -16 & 37 & -29 \end{bmatrix}$$

What is $-3 \cdot F$?

7. Suppose you are given the matrix:

$$E = \begin{bmatrix} 72 & 36 & 28 \\ 84 & 56 & 96 \end{bmatrix}$$

What is $\frac{1}{4} \cdot E$?

8. Suppose you are given the matrix:

$$F = \begin{bmatrix} 24 & -39 & -18 \\ -15 & 33 & -21 \end{bmatrix}$$

What is $-\frac{1}{3} \cdot F$?

9. Suppose you are given two matrices:

$$A = \begin{bmatrix} 3 & 5 & 2 \\ 2 & 8 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 6 & 2 \\ 6 & 1 & 3 \end{bmatrix}$$

What is $4A + 5B$?

10. Suppose you are given two matrices:

$$A = \begin{bmatrix} 3 & 5 & 2 \\ 2 & 8 & 9 \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 & 6 & 8 \\ 6 & 1 & 7 \end{bmatrix}$$

What is $6A - 2B$?

11. Suppose you are given two matrices:

$$A = \begin{bmatrix} 12 & -16 & 24 \\ 14 & 8 & -18 \end{bmatrix} \text{ and } B = \begin{bmatrix} 9 & 6 & 3 \\ 12 & 15 & 18 \end{bmatrix}$$

What is $\frac{1}{2}A + \frac{2}{3}B$?

12. Suppose you are given two matrices:

$$A = \begin{bmatrix} 12 & 28 & 8 \\ 24 & 36 & 16 \end{bmatrix} \text{ and } B = \begin{bmatrix} 8 & 16 & 8 \\ 12 & 22 & 14 \end{bmatrix}$$

What is $\frac{3}{4}A - \frac{1}{2}B$?

Answers – Matrix Algebra Tutor - Worksheet 2 – Adding Matrices, Subtracting Matrices and Multiplying Matrices by a Scalar

1. Suppose you are given two matrices:

$$A = \begin{bmatrix} 3 & 5 & 2 \\ 2 & 8 & 9 \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 & 6 & 2 \\ 6 & 1 & 8 \end{bmatrix}$$

What is $A + B$?

When adding two matrices, the matrices must have the same dimension. Both of these matrices are 2 by 3 matrices so they can be added together. When adding matrices, add corresponding elements.

The formula for adding two matrices is:

$$\begin{bmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{bmatrix} + \begin{bmatrix} b_{1,1} & \cdots & b_{1,n} \\ \vdots & \ddots & \vdots \\ b_{n,1} & \cdots & b_{n,n} \end{bmatrix} = \begin{bmatrix} a_{1,1} + b_{1,1} & \cdots & a_{1,n} + b_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} + b_{n,1} & \cdots & a_{n,n} + b_{n,n} \end{bmatrix}$$

Therefore,

$$A + B = \begin{bmatrix} 3 & 5 & 2 \\ 2 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 7 & 6 & 2 \\ 6 & 1 & 8 \end{bmatrix} = \begin{bmatrix} 3+7 & 5+6 & 2+2 \\ 2+6 & 8+1 & 9+8 \end{bmatrix} = \begin{bmatrix} 10 & 11 & 4 \\ 8 & 9 & 17 \end{bmatrix}$$

Answer: $\begin{bmatrix} 10 & 11 & 4 \\ 8 & 9 & 17 \end{bmatrix}$

2. Suppose you are given two matrices:

$$A = \begin{bmatrix} 13 & 25 & 12 \\ 22 & 38 & 19 \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 & 16 & 8 \\ 16 & 21 & 7 \end{bmatrix}$$

What is $A - B$?

When subtracting two matrices, the matrices must have the same dimension. Both of these matrices are 2 by 3 matrices, so they can be subtracted. When subtracting matrices, subtract corresponding elements.

The formula for subtracting two matrices is:

$$\begin{bmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{bmatrix} - \begin{bmatrix} b_{1,1} & \cdots & b_{1,n} \\ \vdots & \ddots & \vdots \\ b_{n,1} & \cdots & b_{n,n} \end{bmatrix} = \begin{bmatrix} a_{1,1} - b_{1,1} & \cdots & a_{1,n} - b_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} - b_{n,1} & \cdots & a_{n,n} - b_{n,n} \end{bmatrix}$$

Therefore,

$$A - B = \begin{bmatrix} 13 & 25 & 12 \\ 22 & 38 & 19 \end{bmatrix} - \begin{bmatrix} 7 & 16 & 8 \\ 16 & 21 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 13 - 7 & 25 - 16 & 12 - 8 \\ 22 - 16 & 38 - 21 & 19 - 7 \end{bmatrix} = \begin{bmatrix} 6 & 9 & 4 \\ 6 & 17 & 12 \end{bmatrix}$$

Answer: $\begin{bmatrix} 6 & 9 & 4 \\ 6 & 17 & 12 \end{bmatrix}$

3. Suppose you are given two matrices:

$$C = \begin{bmatrix} -34 & 25 & -42 \\ 62 & -18 & 29 \end{bmatrix} \text{ and } D = \begin{bmatrix} 17 & 26 & 42 \\ 36 & 31 & 58 \end{bmatrix}$$

What is $C + D$?

When adding two matrices, the matrices must have the same dimension. Both of these matrices are 2 by 3 matrices, so they can be added together. When adding matrices, add corresponding elements.

The formula for adding two matrices is:

$$\begin{bmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{bmatrix} + \begin{bmatrix} b_{1,1} & \cdots & b_{1,n} \\ \vdots & \ddots & \vdots \\ b_{n,1} & \cdots & b_{n,n} \end{bmatrix} = \begin{bmatrix} a_{1,1} + b_{1,1} & \cdots & a_{1,n} + b_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} + b_{n,1} & \cdots & a_{n,n} + b_{n,n} \end{bmatrix}$$

Therefore,

$$C + D = \begin{bmatrix} -34 & 25 & -42 \\ 62 & -18 & 29 \end{bmatrix} + \begin{bmatrix} 17 & 26 & 42 \\ 36 & 31 & 58 \end{bmatrix}$$

$$= \begin{bmatrix} -34 + 17 & 25 + 26 & -42 + 42 \\ 62 + 36 & -18 + 31 & 29 + 58 \end{bmatrix} = \begin{bmatrix} -17 & 51 & 0 \\ 98 & 13 & 87 \end{bmatrix}$$

Answer: $\begin{bmatrix} -17 & 51 & 0 \\ 98 & 13 & 87 \end{bmatrix}$

4. Suppose you are given two matrices:

$$C = \begin{bmatrix} -32 & -51 & 22 \\ 19 & -81 & -39 \end{bmatrix} \text{ and } D = \begin{bmatrix} -7 & -6 & -8 \\ 26 & 31 & 47 \end{bmatrix}$$

What is $C - D$?

When subtracting two matrices, the matrices must have the same dimension. Both of these matrices are 2 by 3 matrices, so they can be subtracted. When subtracting matrices, subtract corresponding elements.

The formula for subtracting two matrices is:

$$\begin{bmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{bmatrix} - \begin{bmatrix} b_{1,1} & \cdots & b_{1,n} \\ \vdots & \ddots & \vdots \\ b_{n,1} & \cdots & b_{n,n} \end{bmatrix} = \begin{bmatrix} a_{1,1} - b_{1,1} & \cdots & a_{1,n} - b_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} - b_{n,1} & \cdots & a_{n,n} - b_{n,n} \end{bmatrix}$$

Therefore,

$$C - D = \begin{bmatrix} -32 & -51 & 22 \\ 19 & -81 & -39 \end{bmatrix} - \begin{bmatrix} -7 & -6 & -8 \\ 26 & 31 & 47 \end{bmatrix}$$

$$= \begin{bmatrix} -32 - (-7) & -51 - (-6) & 22 - (-8) \\ 19 - 26 & -81 - 31 & -39 - 47 \end{bmatrix} = \begin{bmatrix} -25 & -45 & 30 \\ -7 & -112 & -86 \end{bmatrix}$$

Answer: $\begin{bmatrix} -25 & -45 & 30 \\ -7 & -112 & -86 \end{bmatrix}$

5. Suppose you are given the matrix:

$$E = \begin{bmatrix} 7 & 3 & 2 \\ 8 & 5 & 9 \end{bmatrix}$$

What is $6 \cdot E$?

The number 6 in this problem is called a scalar. When multiplying a matrix by a scalar, multiply each element in the matrix by that scalar. The formula looks like this:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}; c \cdot A = c \cdot \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} c \cdot a_{11} & c \cdot a_{12} \\ c \cdot a_{21} & c \cdot a_{22} \end{bmatrix}$$

Therefore, $6 \cdot E = 6 \cdot \begin{bmatrix} 7 & 3 & 2 \\ 8 & 5 & 9 \end{bmatrix} = \begin{bmatrix} 6 \cdot 7 & 6 \cdot 3 & 6 \cdot 2 \\ 6 \cdot 8 & 6 \cdot 5 & 6 \cdot 9 \end{bmatrix} = \begin{bmatrix} 42 & 18 & 12 \\ 48 & 30 & 54 \end{bmatrix}$

Answer: $\begin{bmatrix} 42 & 18 & 12 \\ 48 & 30 & 54 \end{bmatrix}$

6. Suppose you are given the matrix:

$$F = \begin{bmatrix} 17 & 26 & -18 \\ -16 & 37 & -29 \end{bmatrix}$$

What is $-3 \cdot F$?

The number -3 in this problem is called a scalar. When multiplying a matrix by a scalar, multiply each element in the matrix by that scalar. The formula looks like this:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}; c \cdot A = c \cdot \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} = \begin{bmatrix} c \cdot a_{1,1} & c \cdot a_{1,2} \\ c \cdot a_{2,1} & c \cdot a_{2,2} \end{bmatrix}$$

Therefore, $-3 \cdot F = -3 \cdot \begin{bmatrix} 17 & 26 & -18 \\ -16 & 37 & -29 \end{bmatrix} = \begin{bmatrix} -3 \cdot 17 & -3 \cdot 26 & -3 \cdot -18 \\ -3 \cdot -16 & -3 \cdot 37 & -3 \cdot -29 \end{bmatrix}$

$$= \begin{bmatrix} -51 & -78 & 54 \\ 48 & -111 & 87 \end{bmatrix}$$

Answer: $\begin{bmatrix} -51 & -78 & 54 \\ 48 & -111 & 87 \end{bmatrix}$

7. Suppose you are given the matrix:

$$E = \begin{bmatrix} 72 & 36 & 28 \\ 84 & 56 & 96 \end{bmatrix}$$

What is $\frac{1}{4} \cdot E$?

The fraction $\frac{1}{4}$ in this problem is called a scalar. When multiplying a matrix by a scalar, multiply each element in the matrix by that scalar. The formula looks like this:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}; c \cdot A = c \cdot \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} = \begin{bmatrix} c \cdot a_{1,1} & c \cdot a_{1,2} \\ c \cdot a_{2,1} & c \cdot a_{2,2} \end{bmatrix}$$

Therefore, $\frac{1}{4} \cdot E = \frac{1}{4} \cdot \begin{bmatrix} 72 & 36 & 28 \\ 84 & 56 & 96 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} \cdot 72 & \frac{1}{4} \cdot 36 & \frac{1}{4} \cdot 28 \\ \frac{1}{4} \cdot 84 & \frac{1}{4} \cdot 56 & \frac{1}{4} \cdot 96 \end{bmatrix}$

$$= \begin{bmatrix} 18 & 9 & 7 \\ 21 & 14 & 24 \end{bmatrix}$$

Answer: $\begin{bmatrix} 18 & 9 & 7 \\ 21 & 14 & 24 \end{bmatrix}$

8. Suppose you are given the matrix:

$$F = \begin{bmatrix} 24 & -39 & -18 \\ -15 & 33 & -21 \end{bmatrix}$$

What is $-\frac{1}{3} \cdot F$?

The fraction $-\frac{1}{3}$ in this problem is called a scalar. When multiplying a matrix by a scalar, multiply each element in the matrix by that scalar. The formula looks like this:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}; c \cdot A = c \cdot \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} = \begin{bmatrix} c \cdot a_{1,1} & c \cdot a_{1,2} \\ c \cdot a_{2,1} & c \cdot a_{2,2} \end{bmatrix}$$

Therefore,

$$-\frac{1}{3} \cdot F = -\frac{1}{3} \cdot \begin{bmatrix} 24 & -39 & -18 \\ -15 & 33 & -21 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} \cdot 24 & -\frac{1}{3} \cdot -39 & -\frac{1}{3} \cdot -18 \\ -\frac{1}{3} \cdot -15 & -\frac{1}{3} \cdot 33 & -\frac{1}{3} \cdot -21 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 13 & 6 \\ 5 & -11 & 7 \end{bmatrix}$$

Answer: $\begin{bmatrix} -8 & 13 & 6 \\ 5 & -11 & 7 \end{bmatrix}$

9. Suppose you are given two matrices:

$$A = \begin{bmatrix} 3 & 5 & 2 \\ 2 & 8 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 6 & 2 \\ 6 & 1 & 3 \end{bmatrix}$$

What is $4A + 5B$?

Multiply the matrices by their scalars and then add the results.

$$4A = 4 \cdot \begin{bmatrix} 3 & 5 & 2 \\ 2 & 8 & 3 \end{bmatrix} = \begin{bmatrix} 12 & 20 & 8 \\ 8 & 32 & 12 \end{bmatrix}$$

$$5B = 5 \cdot \begin{bmatrix} 3 & 6 & 2 \\ 6 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 15 & 30 & 10 \\ 30 & 5 & 15 \end{bmatrix}$$

$$4A + 5B = \begin{bmatrix} 12 & 20 & 8 \\ 8 & 32 & 12 \end{bmatrix} + \begin{bmatrix} 15 & 30 & 10 \\ 30 & 5 & 15 \end{bmatrix} = \begin{bmatrix} 27 & 50 & 18 \\ 38 & 37 & 27 \end{bmatrix}$$

Answer: $\begin{bmatrix} 27 & 50 & 18 \\ 38 & 37 & 27 \end{bmatrix}$

10. Suppose you are given two matrices:

$$A = \begin{bmatrix} 3 & 5 & 2 \\ 2 & 8 & 9 \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 & 6 & 8 \\ 6 & 1 & 7 \end{bmatrix}$$

What is $6A - 2B$?

Multiply the matrices by their scalars and then subtract the results.

$$6A = 6 \cdot \begin{bmatrix} 3 & 5 & 2 \\ 2 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 18 & 30 & 12 \\ 12 & 48 & 54 \end{bmatrix}$$

$$2B = 2 \cdot \begin{bmatrix} 7 & 6 & 8 \\ 6 & 1 & 7 \end{bmatrix} = \begin{bmatrix} 14 & 12 & 16 \\ 12 & 2 & 14 \end{bmatrix}$$

$$6A - 2B = \begin{bmatrix} 18 & 30 & 12 \\ 12 & 48 & 54 \end{bmatrix} - \begin{bmatrix} 14 & 12 & 16 \\ 12 & 2 & 14 \end{bmatrix} = \begin{bmatrix} 4 & 18 & -4 \\ 0 & 46 & 40 \end{bmatrix}$$

Answer: $\begin{bmatrix} 4 & 18 & -4 \\ 0 & 46 & 40 \end{bmatrix}$

11. Suppose you are given two matrices:

$$A = \begin{bmatrix} 12 & -16 & 24 \\ 14 & 8 & -18 \end{bmatrix} \text{ and } B = \begin{bmatrix} 9 & 6 & 3 \\ 12 & 15 & 18 \end{bmatrix}$$

What is $\frac{1}{2}A + \frac{2}{3}B$?

Multiply the matrices by their scalars and then add the results.

$$\frac{1}{2}A = \frac{1}{2} \cdot \begin{bmatrix} 12 & -16 & 24 \\ 14 & 8 & -18 \end{bmatrix} = \begin{bmatrix} 6 & -8 & 12 \\ 7 & 4 & -9 \end{bmatrix}$$

$$\frac{2}{3}B = \frac{2}{3} \cdot \begin{bmatrix} 9 & 6 & 3 \\ 12 & 15 & 18 \end{bmatrix} = \begin{bmatrix} 6 & 4 & 2 \\ 8 & 10 & 12 \end{bmatrix}$$

$$\frac{1}{2}A + \frac{2}{3}B = \begin{bmatrix} 6 & -8 & 12 \\ 7 & 4 & -9 \end{bmatrix} + \begin{bmatrix} 6 & 4 & 2 \\ 8 & 10 & 12 \end{bmatrix} = \begin{bmatrix} 12 & -4 & 14 \\ 15 & 14 & 3 \end{bmatrix}$$

Answer: $\begin{bmatrix} 12 & -4 & 14 \\ 15 & 14 & 3 \end{bmatrix}$

12. Suppose you are given two matrices:

$$A = \begin{bmatrix} 12 & 28 & 8 \\ 24 & 36 & 16 \end{bmatrix} \text{ and } B = \begin{bmatrix} 8 & 16 & 8 \\ 12 & 22 & 14 \end{bmatrix}$$

What is $\frac{3}{4}A - \frac{1}{2}B$?

Multiply the matrices by their scalars and then subtract the results.

$$\frac{3}{4}A = \frac{3}{4} \cdot \begin{bmatrix} 12 & 28 & 8 \\ 24 & 36 & 16 \end{bmatrix} = \begin{bmatrix} 9 & 21 & 6 \\ 18 & 27 & 12 \end{bmatrix}$$

$$\frac{1}{2}B = \frac{1}{2} \cdot \begin{bmatrix} 8 & 16 & 8 \\ 12 & 22 & 14 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 4 \\ 6 & 11 & 7 \end{bmatrix}$$

$$\frac{3}{4}A - \frac{1}{2}B = \begin{bmatrix} 9 & 21 & 6 \\ 18 & 27 & 12 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 4 \\ 6 & 11 & 7 \end{bmatrix} = \begin{bmatrix} 5 & 13 & 2 \\ 12 & 16 & 5 \end{bmatrix}$$

Answer: $\begin{bmatrix} 5 & 13 & 2 \\ 12 & 16 & 5 \end{bmatrix}$